Math 21b Additional Midterm Practice Questions
Petko's Section – October 30, 2005

True/False Questions:
1. If the 5 x 5 matrix A has rank 5, then any linear system with coefficient matrix A will have a unique solution.
T   F

2. If A is an n x n matrix and $\vec{x}$ is a vector in $\Re^n$, then the product $A\,\vec{x}$ is a linear combination of the column vectors of the matrix A.
T   F

3. If A is a nonzero matrix of the form $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then rank$(A) = 2$.

T   F

4. If $\vec{u}$, $\vec{v}$, and $\vec{w}$ are vectors in $\Re^2$, then $\vec{w}$ must be a linear combination of $\vec{u}$ and $\vec{v}$.
T   F

5. A linear system with fewer unknowns than equations must have infinitely many solutions or none.
T   F

6. If A is a 4 x 3 matrix of rank 3 and $A\,\vec{v} = A\,\vec{w}$ for two vectors $\vec{v}$ and $\vec{w}$ in $\Re^3$, then the vectors $\vec{v}$ and $\vec{w}$ must be equal.
T   F

7. There exists a 4 x 3 matrix A of rank 3 such that $A \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \vec{0}$.

T   F

8. The linear system $A\,\vec{x} = \vec{b}$ is consistent if (and only if) rank$(A) = $ rank$[\,A \vdots \vec{b}\,]$.
T   F

9. Two matrices A and B have the same reduced row-echelon form if (and only if) the equations $A\,\vec{x} = \vec{0}$ and $B\,\vec{x} = \vec{0}$ have the same solutions.
T   F

10. If A is an invertible 2 x 2 matrix and B is any 2 x 2 matrix, then the formula rank(AB) = rank(B) must hold.
T   F

11. If a matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ represents the orthogonal projection onto a line L, then the equation $a^2 + b^2 + c^2 + d^2 = 1$ must hold.

T   F

12. There exists an invertible 9 x 9 matrix that has 74 ones among its entries.

T   F

13. If A is an n x n matrix such that $A^2 = 0$, then the matrix $I_n + A$ must be invertible.

T   F

14. There exists a 2 x 3 matrix A and a 3 x 2 matrix B such that $AB = I_2$.

T   F

15. Two j x k matrices, A and B, are equal if (and only if) $A\vec{x} = B\vec{x}$ for all $\vec{x} \in \Re^k$.

T   F

16. There exists a positive integer n such that $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}^n = I_2$.

T   F

17. The matrix $\begin{bmatrix} 5 & 6 \\ -6 & 5 \end{bmatrix}$ represents a rotation combined with scaling.

T   F

18. If $A^3\vec{x} = \vec{0}$ but $A^2\vec{x} \neq \vec{0}$, then $A^2\vec{x}, A\vec{x}$, and $\vec{x}$ are linearly independent.

T   F

19. If A is an n x n matrix such that $A^2 = A$, then the image of A and the kernel of A have only the zero vector in common.

T   F

20. If the 2 x 2 matrix A represents a reflection about the line L in $\Re^2$, then A must be similar to the matrix $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

T   F

21. For every subspace V of $\Re^3$ there exists a 3 x 3 matrix A such that $V = \text{im}(A)$.

T   F

22. If A is similar to B, and A is invertible, then B must be invertible as well.

T   F

23. There exists a 2 x 2 matrix A such that $im(A) = ker(A)$.

T   F

24. If V and W are subspaces of $\mathfrak{R}^n$, then their union $V \cup W$ must be a subspace of $\mathfrak{R}^n$ as well.

T   F

25. If A is an invertible n x n matrix, then the kernels of A and A^{-1} must equal.

T   F

26. If two vectors are linearly dependent, then one must be a scalar multiple of the other.

T   F

27. If $\vec{x}$ is in the kernel of B, then, if the matrix product AB is defined, $\vec{x} \in ker(AB)$.

T   F

28. The matrix $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ is similar to $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

T   F

29. Vectors of the form $\begin{bmatrix} a \\ 0 \\ b \\ b \end{bmatrix}$ (where a and b are arbitrary) form a subspace of $\mathfrak{R}^4$.

T   F

30. If A is similar to B, and B is similar to C, then C must be similar to A.

T   F

31. If $ker(A) = \vec{0}$, then the column vectors of A must be linearly independent and the matrix A is invertible.

T   F

32. If $\vec{u}$ is a unit vector in $\mathfrak{R}^n$, and $L = span(\vec{u})$, then $proj_L(\vec{y}) = (\vec{y} \cdot \vec{u})\vec{y}$ for all $\vec{y}$ in $\mathfrak{R}^n$.

T   F

33. There exists a subspace V of $\mathfrak{R}^5$ such that $dim(V) = dim(V^\perp)$.

T   F

34. If $\vec{x}$ and $\vec{y}$ are vectors in $\mathfrak{R}^n$, then $\|\vec{x} + \vec{y}\| = \|\vec{x}\|^2 + \|\vec{y}\|^2$.

T   F

35. For any q, the vectors $\begin{bmatrix} \cos q \\ \sin q \end{bmatrix}$ and $\begin{bmatrix} -\sin q \\ \cos q \end{bmatrix}$ form an orthonormal basis of $\mathfrak{R}^2$.

T   F

36. If A is an n x m matrix and $\vec{b}$ is a nonzero vector in $\mathfrak{R}^m$, then the set of solutions to the equation $A\vec{x} = \vec{b}$ form a subspace of $\mathfrak{R}^n$.

T   F

Skills questions:

1) Write the matrix with respect to the standard basis of the reflection about the line $3x - y = 0$.

2) Find the orthogonal projection of $\begin{bmatrix} 49 \\ 49 \\ 49 \end{bmatrix}$ onto the subspace $\mathrm{span}\left\{ \begin{bmatrix} 2 \\ 3 \\ 6 \end{bmatrix}, \begin{bmatrix} 3 \\ -6 \\ 2 \end{bmatrix} \right\}$.

3) Given that $\mathrm{rref}(A) = \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$, find the dimensions of $\ker(A)$ and $\mathrm{im}(A)$. Find a basis for $\ker(A)$ and $\mathrm{im}(A)$.